“Single Machine Scheduling with PuLP”

Ahmet Yiğit Doğan

12.03.2022

Table of Contents

Implicit Model

Part i)

$$Sets: \quad J = \{1, 2, 3, 4, 5\}$$

$$Parameters: \quad p_j = \{2, 2, 1, 3, 4\}$$
$$d_j = \{3, 3, 1, 8, 5\}$$
$$T = 12$$

$$Decision\ variables: \quad x_{j,t} \begin{cases} 1 & if\ the\ job\ j\ starts\ at\ the\ time\ t \\ 0 & otherwise \end{cases}$$

$$min \sum_{j=1}^{J} \sum_{t=d_j-p_j+1}^{T-p_j} x_{j,t}$$

(For each job, starting time should not exceed the latest index in which the completion in its due date is possible: $d_j - p_j + 1$. The final starting dates that each job can be started is $T - p_j$. See *appendix 1.1* for the explicit formulation.)

$$st. \sum_{t=1}^{T-p_j+1} x_{j,t} = 1 \quad for\ \forall j \in J$$

(Every job must start exactly once within the possible period it can be completed. See *appendix 1.2* for the explicit formulation.)

$$x_{j,t} + x_{j',t'} \leq 1 \quad for\ \forall j \in J$$
$$j' \in J \setminus \{j\}$$
$$t = 1, 2 \ldots T, t' = 1, 2 \ldots T$$
$$if\ (j,t)\ and\ (j',t')\ overlap$$

(Possible processing periods of each pair of jobs must not overlap. See *appendix 1.3* for the explicit formulation.)

Additionally, all overlap conditions can be detected by the following way:

$- x_{j,t}$ *states whether the job* j *starts at the time* t, *and consequently, ends at the time* $t + p_j$.
$- x_{j',t'}$ *states whether the job* j' *starts at the time* t', *and ends at the time* $t' + p_{j'}$.

$$- if \begin{cases} [t - (t' + p_{j'})] \times [(t + p_j) - t'] < 0, & x_{j,t}\ and\ x_{j',t'}\ overlap \\ otherwise, & x_{j,t}\ and\ x_{j',t'}\ do\ not\ overlap \end{cases}$$

Finally, $x_{j,t}$ values should be restricted to be binary:

$$x_{j,t} \in \{0,1\} \quad for\ \forall j \in J,$$
$$t = \{1, 2, \ldots T\}$$

(See *appendix 1.5* to see the variables. During the computations in the notebook, they are changed as $0 \leq x_{j,t} \leq 1$, and the unnecessary ones, which force the time horizon to be longer than 12 are dropped as well.)

Part ii)

$$Sets: \qquad J = \{1, 2, 3, 4, 5\}$$

$$Parameters: \quad p_j = \{2, 2, 1, 3, 4\}$$
$$d_j = \{3, 3, 1, 8, 5\}$$
$$T = 12$$

$$Decision\ variables: \quad x_{j,t} \begin{cases} 1 & if\ the\ job\ j\ starts\ at\ the\ time\ t \\ 0 & otherwise \end{cases}$$

$$min \sum_{j=1}^{J} \sum_{t=d_j-p_j+1}^{T-p_j} x_{j,t}$$

$$st. \sum_{t=1}^{T-p_j+1} x_{j,t} = 1 \quad for\ \forall j \in J$$

$$\sum_{j=1}^{J} \sum_{s=\max\{0,\ t-p_j+1\}}^{t} x_{j,t} \leq 1 \quad for\ t = 1, 2, \ldots T$$

(For each time period t, and for each job j, the case of being processed at that time must not overlap. See *appendix 1.4* for the explicit formulation.)

$$x_{j,t} \in \{0,1\} \quad for\ \forall j \in J,$$
$$t = \{1, 2, \ldots T\}$$

Computation Times

As can be expected and clearly observed from the table below, time-based approach ($Complexity$: $O(T)$) is a better representation of the problem in terms of efficiency compared to the pairwise consideration of each possible job initialization case ($O(J^2, T^2)$).

Environment	Jupyter Notebook	Google Colab	Visual Studio Code
Model i	0,069	0,105	0,051
Model ii	0,046	0,031	0,022

Table 1: Computation Times (in Seconds)

The Optimal Solution

The two models above have given the same optimal value for the minimum number of tardy jobs, with different sequences of the time horizon. The solutions obtained after the execution of the program are as follows:

Model i:

$z^* = 2$
$x_{1,1} = 1$
$x_{2,6} = 1$
$x_{3,0} = 1$
$x_{4,3} = 1$
$x_{5,8} = 1$

Model_ii:

$z^* = 2$
$x_{1,3} = 1$
$x_{2,1} = 1$
$x_{3,0} = 1$
$x_{4,5} = 1$
$x_{5,8} = 1$

Where all the remaining decision variables are equal to zero.

Time Periods	**0 - 1**	**1 - 2**	**2 - 3**	**3 - 4**	**4 - 5**	**5 - 6**	**6 - 7**	**7 - 8**	**8 - 9**	**9 - 10**	**10 - 11**	**11 - 12**
Job # (Model 1)	3	1	1	4	4	4	2	2	5	5	5	5
Job # (Model 2)	3	2	2	1	1	4	4	4	5	5	5	5

Table 2: The Final States of the Time Horizon

Appendices

1.1. Objective Function

$$\min \quad (x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} + x_{1,7} + x_{1,8} + x_{1,9} + x_{1,10} +$$
$$x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} + x_{2,6} + x_{2,7} + x_{2,8} + x_{2,9} + x_{2,10} +$$
$$x_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} + x_{3,5} + x_{3,6} + x_{3,7} + x_{3,8} + x_{3,9} + x_{3,10} + x_{3,11} +$$
$$x_{4,6} + x_{4,7} + x_{4,8} + x_{4,9} +$$
$$x_{5,2} + x_{5,3} + x_{5,4} + x_{5,5} + x_{5,6} + x_{5,7} + x_{5,8})$$

1.2. Type 1 Constraints for the Parts i and ii

$$x_{1,0} + x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} + x_{1,7} + x_{1,8} + x_{1,9} + x_{1,10} = 1$$
$$x_{2,0} + x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} + x_{2,5} + x_{2,6} + x_{2,7} + x_{2,8} + x_{2,9} + x_{2,10} = 1$$
$$x_{3,0} + x_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} + x_{3,5} + x_{3,6} + x_{3,7} + x_{3,8} + x_{3,9} + x_{3,10} + x_{3,11} = 1$$
$$x_{4,0} + x_{4,1} + x_{4,2} + x_{4,3} + x_{4,4} + x_{4,5} + x_{4,6} + x_{4,7} + x_{4,8} + x_{4,9} = 1$$
$$x_{5,0} + x_{5,1} + x_{5,2} + x_{5,3} + x_{5,4} + x_{5,5} + x_{5,6} + x_{5,7} + x_{5,8} = 1$$

1.3. Type 2 Constraints for the Part i

$x_{1,0} + x_{2,0} \le 1$
$x_{1,0} + x_{2,1} \le 1$
$x_{1,0} + x_{3,0} \le 1$
$x_{1,0} + x_{3,1} \le 1$
$x_{1,0} + x_{4,0} \le 1$
$x_{1,0} + x_{4,1} \le 1$
$x_{1,0} + x_{5,0} \le 1$
$x_{1,0} + x_{5,1} \le 1$
$x_{1,1} + x_{2,0} \le 1$
$x_{1,1} + x_{2,1} \le 1$
$x_{1,1} + x_{2,2} \le 1$
$x_{1,1} + x_{3,1} \le 1$
$x_{1,1} + x_{3,2} \le 1$
$x_{1,1} + x_{4,0} \le 1$
$x_{1,1} + x_{4,1} \le 1$
$x_{1,1} + x_{4,2} \le 1$
$x_{1,1} + x_{5,0} \le 1$
$x_{1,1} + x_{5,1} \le 1$
$x_{1,1} + x_{5,2} \le 1$
$x_{1,2} + x_{2,1} \le 1$
$x_{1,2} + x_{2,2} \le 1$
$x_{1,2} + x_{2,3} \le 1$
$x_{1,2} + x_{3,2} \le 1$
$x_{1,2} + x_{3,3} \le 1$
$x_{1,2} + x_{4,0} \le 1$
$x_{1,2} + x_{4,1} \le 1$
$x_{1,2} + x_{4,2} \le 1$
$x_{1,2} + x_{4,3} \le 1$
$x_{1,2} + x_{5,0} \le 1$
$x_{1,2} + x_{5,1} \le 1$
$x_{1,2} + x_{5,2} \le 1$
$x_{1,2} + x_{5,3} \le 1$
$x_{1,3} + x_{2,2} \le 1$
$x_{1,3} + x_{2,3} \le 1$
$x_{1,3} + x_{2,4} \le 1$
$x_{1,3} + x_{3,3} \le 1$
$x_{1,3} + x_{3,4} \le 1$
$x_{1,3} + x_{4,1} \le 1$
$x_{1,3} + x_{4,2} \le 1$
$x_{1,3} + x_{4,3} \le 1$
$x_{1,3} + x_{4,4} \le 1$
$x_{1,3} + x_{5,0} \le 1$
$x_{1,3} + x_{5,1} \le 1$
$x_{1,3} + x_{5,2} \le 1$
$x_{1,3} + x_{5,3} \le 1$
$x_{1,3} + x_{5,4} \le 1$
$x_{1,4} + x_{2,3} \le 1$
$x_{1,4} + x_{2,4} \le 1$
$x_{1,4} + x_{2,5} \le 1$
$x_{1,4} + x_{3,4} \le 1$
$x_{1,4} + x_{3,5} \le 1$
$x_{1,4} + x_{4,2} \le 1$
$x_{1,4} + x_{4,3} \le 1$
$x_{1,4} + x_{4,4} \le 1$
$x_{1,4} + x_{4,5} \le 1$
$x_{1,4} + x_{5,1} \le 1$
$x_{1,4} + x_{5,2} \le 1$
$x_{1,4} + x_{5,3} \le 1$
$x_{1,4} + x_{5,4} \le 1$
$x_{1,4} + x_{5,5} \le 1$
$x_{1,5} + x_{2,4} \le 1$
$x_{1,5} + x_{2,5} \le 1$
$x_{1,5} + x_{2,6} \le 1$
$x_{1,5} + x_{3,5} \le 1$
$x_{1,5} + x_{3,6} \le 1$
$x_{1,5} + x_{4,3} \le 1$
$x_{1,5} + x_{4,4} \le 1$
$x_{1,5} + x_{4,5} \le 1$
$x_{1,5} + x_{4,6} \le 1$
$x_{1,5} + x_{5,2} \le 1$
$x_{1,5} + x_{5,3} \le 1$
$x_{1,5} + x_{5,4} \le 1$
$x_{1,5} + x_{5,5} \le 1$
$x_{1,5} + x_{5,6} \le 1$
$x_{1,6} + x_{2,5} \le 1$
$x_{1,6} + x_{2,6} \le 1$
$x_{1,6} + x_{2,7} \le 1$
$x_{1,6} + x_{3,6} \le 1$
$x_{1,6} + x_{3,7} \le 1$
$x_{1,6} + x_{4,4} \le 1$
$x_{1,6} + x_{4,5} \le 1$
$x_{1,6} + x_{4,6} \le 1$
$x_{1,6} + x_{4,7} \le 1$
$x_{1,6} + x_{5,3} \le 1$
$x_{1,6} + x_{5,4} \le 1$
$x_{1,6} + x_{5,5} \le 1$
$x_{1,6} + x_{5,6} \le 1$
$x_{1,6} + x_{5,7} \le 1$
$x_{1,7} + x_{2,6} \le 1$
$x_{1,7} + x_{2,7} \le 1$

$x_{1,7} + x_{2,8} \leq 1$
$x_{1,7} + x_{3,7} \leq 1$
$x_{1,7} + x_{3,8} \leq 1$
$x_{1,7} + x_{4,5} \leq 1$
$x_{1,7} + x_{4,6} \leq 1$
$x_{1,7} + x_{4,7} \leq 1$
$x_{1,7} + x_{4,8} \leq 1$
$x_{1,7} + x_{5,4} \leq 1$
$x_{1,7} + x_{5,5} \leq 1$
$x_{1,7} + x_{5,6} \leq 1$
$x_{1,7} + x_{5,7} \leq 1$
$x_{1,7} + x_{5,8} \leq 1$
$x_{1,8} + x_{2,7} \leq 1$
$x_{1,8} + x_{2,8} \leq 1$
$x_{1,8} + x_{2,9} \leq 1$
$x_{1,8} + x_{3,8} \leq 1$
$x_{1,8} + x_{3,9} \leq 1$
$x_{1,8} + x_{4,6} \leq 1$
$x_{1,8} + x_{4,7} \leq 1$
$x_{1,8} + x_{4,8} \leq 1$
$x_{1,8} + x_{4,9} \leq 1$
$x_{1,8} + x_{5,5} \leq 1$
$x_{1,8} + x_{5,6} \leq 1$
$x_{1,8} + x_{5,7} \leq 1$
$x_{1,8} + x_{5,8} \leq 1$
$x_{1,9} + x_{2,8} \leq 1$
$x_{1,9} + x_{2,9} \leq 1$
$x_{1,9} + x_{2,10} \leq 1$
$x_{1,9} + x_{3,9} \leq 1$
$x_{1,9} + x_{3,10} \leq 1$
$x_{1,9} + x_{4,7} \leq 1$
$x_{1,9} + x_{4,8} \leq 1$
$x_{1,9} + x_{4,9} \leq 1$
$x_{1,9} + x_{5,6} \leq 1$
$x_{1,9} + x_{5,7} \leq 1$
$x_{1,9} + x_{5,8} \leq 1$
$x_{1,10} + x_{2,9} \leq 1$
$x_{1,10} + x_{2,10} \leq 1$
$x_{1,10} + x_{3,10} \leq 1$
$x_{1,10} + x_{3,11} \leq 1$
$x_{1,10} + x_{4,8} \leq 1$
$x_{1,10} + x_{4,9} \leq 1$
$x_{1,10} + x_{5,7} \leq 1$
$x_{1,10} + x_{5,8} \leq 1$
$x_{2,0} + x_{3,0} \leq 1$
$x_{2,0} + x_{3,1} \leq 1$
$x_{2,0} + x_{4,0} \leq 1$
$x_{2,0} + x_{4,1} \leq 1$
$x_{2,0} + x_{5,0} \leq 1$
$x_{2,0} + x_{5,1} \leq 1$
$x_{2,1} + x_{3,1} \leq 1$

$x_{2,1} + x_{3,2} \leq 1$
$x_{2,1} + x_{4,0} \leq 1$
$x_{2,1} + x_{4,1} \leq 1$
$x_{2,1} + x_{4,2} \leq 1$
$x_{2,1} + x_{5,0} \leq 1$
$x_{2,1} + x_{5,1} \leq 1$
$x_{2,1} + x_{5,2} \leq 1$
$x_{2,2} + x_{3,2} \leq 1$
$x_{2,2} + x_{3,3} \leq 1$
$x_{2,2} + x_{4,0} \leq 1$
$x_{2,2} + x_{4,1} \leq 1$
$x_{2,2} + x_{4,2} \leq 1$
$x_{2,2} + x_{4,3} \leq 1$
$x_{2,2} + x_{5,0} \leq 1$
$x_{2,2} + x_{5,1} \leq 1$
$x_{2,2} + x_{5,2} \leq 1$
$x_{2,2} + x_{5,3} \leq 1$
$x_{2,3} + x_{3,3} \leq 1$
$x_{2,3} + x_{3,4} \leq 1$
$x_{2,3} + x_{4,1} \leq 1$
$x_{2,3} + x_{4,2} \leq 1$
$x_{2,3} + x_{4,3} \leq 1$
$x_{2,3} + x_{4,4} \leq 1$
$x_{2,3} + x_{5,0} \leq 1$
$x_{2,3} + x_{5,1} \leq 1$
$x_{2,3} + x_{5,2} \leq 1$
$x_{2,3} + x_{5,3} \leq 1$
$x_{2,3} + x_{5,4} \leq 1$
$x_{2,4} + x_{3,4} \leq 1$
$x_{2,4} + x_{3,5} \leq 1$
$x_{2,4} + x_{4,2} \leq 1$
$x_{2,4} + x_{4,3} \leq 1$
$x_{2,4} + x_{4,4} \leq 1$
$x_{2,4} + x_{4,5} \leq 1$
$x_{2,4} + x_{5,1} \leq 1$
$x_{2,4} + x_{5,2} \leq 1$
$x_{2,4} + x_{5,3} \leq 1$
$x_{2,4} + x_{5,4} \leq 1$
$x_{2,4} + x_{5,5} \leq 1$
$x_{2,5} + x_{3,5} \leq 1$
$x_{2,5} + x_{3,6} \leq 1$
$x_{2,5} + x_{4,3} \leq 1$
$x_{2,5} + x_{4,4} \leq 1$
$x_{2,5} + x_{4,5} \leq 1$
$x_{2,5} + x_{4,6} \leq 1$
$x_{2,5} + x_{5,2} \leq 1$
$x_{2,5} + x_{5,3} \leq 1$
$x_{2,5} + x_{5,4} \leq 1$
$x_{2,5} + x_{5,5} \leq 1$
$x_{2,5} + x_{5,6} \leq 1$
$x_{2,6} + x_{3,6} \leq 1$

$x_{2,6} + x_{3,7} \leq 1$
$x_{2,6} + x_{4,4} \leq 1$
$x_{2,6} + x_{4,5} \leq 1$
$x_{2,6} + x_{4,6} \leq 1$
$x_{2,6} + x_{4,7} \leq 1$
$x_{2,6} + x_{5,3} \leq 1$
$x_{2,6} + x_{5,4} \leq 1$
$x_{2,6} + x_{5,5} \leq 1$
$x_{2,6} + x_{5,6} \leq 1$
$x_{2,6} + x_{5,7} \leq 1$
$x_{2,7} + x_{3,7} \leq 1$
$x_{2,7} + x_{3,8} \leq 1$
$x_{2,7} + x_{4,5} \leq 1$
$x_{2,7} + x_{4,6} \leq 1$
$x_{2,7} + x_{4,7} \leq 1$
$x_{2,7} + x_{4,8} \leq 1$
$x_{2,7} + x_{5,4} \leq 1$
$x_{2,7} + x_{5,5} \leq 1$
$x_{2,7} + x_{5,6} \leq 1$
$x_{2,7} + x_{5,7} \leq 1$
$x_{2,7} + x_{5,8} \leq 1$
$x_{2,8} + x_{3,8} \leq 1$
$x_{2,8} + x_{3,9} \leq 1$
$x_{2,8} + x_{4,6} \leq 1$
$x_{2,8} + x_{4,7} \leq 1$
$x_{2,8} + x_{4,8} \leq 1$
$x_{2,8} + x_{4,9} \leq 1$
$x_{2,8} + x_{5,5} \leq 1$
$x_{2,8} + x_{5,6} \leq 1$
$x_{2,8} + x_{5,7} \leq 1$
$x_{2,8} + x_{5,8} \leq 1$
$x_{2,9} + x_{3,9} \leq 1$
$x_{2,9} + x_{3,10} \leq 1$
$x_{2,9} + x_{4,7} \leq 1$
$x_{2,9} + x_{4,8} \leq 1$
$x_{2,9} + x_{4,9} \leq 1$
$x_{2,9} + x_{5,6} \leq 1$
$x_{2,9} + x_{5,7} \leq 1$
$x_{2,9} + x_{5,8} \leq 1$
$x_{2,10} + x_{3,10} \leq 1$
$x_{2,10} + x_{3,11} \leq 1$
$x_{2,10} + x_{4,8} \leq 1$
$x_{2,10} + x_{4,9} \leq 1$
$x_{2,10} + x_{5,7} \leq 1$
$x_{2,10} + x_{5,8} \leq 1$
$x_{3,0} + x_{4,0} \leq 1$
$x_{3,0} + x_{5,0} \leq 1$
$x_{3,1} + x_{4,0} \leq 1$
$x_{3,1} + x_{4,1} \leq 1$
$x_{3,1} + x_{5,0} \leq 1$
$x_{3,1} + x_{5,1} \leq 1$

$x_{3,2} + x_{4,0} \le 1$
$x_{3,2} + x_{4,1} \le 1$
$x_{3,2} + x_{4,2} \le 1$
$x_{3,2} + x_{5,0} \le 1$
$x_{3,2} + x_{5,1} \le 1$
$x_{3,2} + x_{5,2} \le 1$
$x_{3,3} + x_{4,1} \le 1$
$x_{3,3} + x_{4,2} \le 1$
$x_{3,3} + x_{4,3} \le 1$
$x_{3,3} + x_{5,0} \le 1$
$x_{3,3} + x_{5,1} \le 1$
$x_{3,3} + x_{5,2} \le 1$
$x_{3,3} + x_{5,3} \le 1$
$x_{3,4} + x_{4,2} \le 1$
$x_{3,4} + x_{4,3} \le 1$
$x_{3,4} + x_{4,4} \le 1$
$x_{3,4} + x_{5,1} \le 1$
$x_{3,4} + x_{5,2} \le 1$
$x_{3,4} + x_{5,3} \le 1$
$x_{3,4} + x_{5,4} \le 1$
$x_{3,5} + x_{4,3} \le 1$
$x_{3,5} + x_{4,4} \le 1$
$x_{3,5} + x_{4,5} \le 1$
$x_{3,5} + x_{5,2} \le 1$
$x_{3,5} + x_{5,3} \le 1$
$x_{3,5} + x_{5,4} \le 1$
$x_{3,5} + x_{5,5} \le 1$
$x_{3,6} + x_{4,4} \le 1$
$x_{3,6} + x_{4,5} \le 1$
$x_{3,6} + x_{4,6} \le 1$
$x_{3,6} + x_{5,3} \le 1$
$x_{3,6} + x_{5,4} \le 1$
$x_{3,6} + x_{5,5} \le 1$
$x_{3,6} + x_{5,6} \le 1$
$x_{3,7} + x_{4,5} \le 1$
$x_{3,7} + x_{4,6} \le 1$

$x_{3,7} + x_{4,7} \le 1$
$x_{3,7} + x_{5,4} \le 1$
$x_{3,7} + x_{5,5} \le 1$
$x_{3,7} + x_{5,6} \le 1$
$x_{3,7} + x_{5,7} \le 1$
$x_{3,8} + x_{4,6} \le 1$
$x_{3,8} + x_{4,7} \le 1$
$x_{3,8} + x_{4,8} \le 1$
$x_{3,8} + x_{5,5} \le 1$
$x_{3,8} + x_{5,6} \le 1$
$x_{3,8} + x_{5,7} \le 1$
$x_{3,8} + x_{5,8} \le 1$
$x_{3,9} + x_{4,7} \le 1$
$x_{3,9} + x_{4,8} \le 1$
$x_{3,9} + x_{4,9} \le 1$
$x_{3,9} + x_{5,6} \le 1$
$x_{3,9} + x_{5,7} \le 1$
$x_{3,9} + x_{5,8} \le 1$
$x_{3,10} + x_{4,8} \le 1$
$x_{3,10} + x_{4,9} \le 1$
$x_{3,10} + x_{5,7} \le 1$
$x_{3,10} + x_{5,8} \le 1$
$x_{3,11} + x_{4,9} \le 1$
$x_{3,11} + x_{5,8} \le 1$
$x_{4,0} + x_{5,0} \le 1$
$x_{4,0} + x_{5,1} \le 1$
$x_{4,0} + x_{5,2} \le 1$
$x_{4,1} + x_{5,0} \le 1$
$x_{4,1} + x_{5,1} \le 1$
$x_{4,1} + x_{5,2} \le 1$
$x_{4,1} + x_{5,3} \le 1$
$x_{4,2} + x_{5,0} \le 1$
$x_{4,2} + x_{5,1} \le 1$
$x_{4,2} + x_{5,2} \le 1$
$x_{4,2} + x_{5,3} \le 1$
$x_{4,2} + x_{5,4} \le 1$

$x_{4,3} + x_{5,0} \le 1$
$x_{4,3} + x_{5,1} \le 1$
$x_{4,3} + x_{5,2} \le 1$
$x_{4,3} + x_{5,3} \le 1$
$x_{4,3} + x_{5,4} \le 1$
$x_{4,3} + x_{5,5} \le 1$
$x_{4,4} + x_{5,1} \le 1$
$x_{4,4} + x_{5,2} \le 1$
$x_{4,4} + x_{5,3} \le 1$
$x_{4,4} + x_{5,4} \le 1$
$x_{4,4} + x_{5,5} \le 1$
$x_{4,4} + x_{5,6} \le 1$
$x_{4,5} + x_{5,2} \le 1$
$x_{4,5} + x_{5,3} \le 1$
$x_{4,5} + x_{5,4} \le 1$
$x_{4,5} + x_{5,5} \le 1$
$x_{4,5} + x_{5,6} \le 1$
$x_{4,5} + x_{5,7} \le 1$
$x_{4,6} + x_{5,3} \le 1$
$x_{4,6} + x_{5,4} \le 1$
$x_{4,6} + x_{5,5} \le 1$
$x_{4,6} + x_{5,6} \le 1$
$x_{4,6} + x_{5,7} \le 1$
$x_{4,6} + x_{5,8} \le 1$
$x_{4,7} + x_{5,4} \le 1$
$x_{4,7} + x_{5,5} \le 1$
$x_{4,7} + x_{5,6} \le 1$
$x_{4,7} + x_{5,7} \le 1$
$x_{4,7} + x_{5,8} \le 1$
$x_{4,8} + x_{5,5} \le 1$
$x_{4,8} + x_{5,6} \le 1$
$x_{4,8} + x_{5,7} \le 1$
$x_{4,8} + x_{5,8} \le 1$
$x_{4,9} + x_{5,6} \le 1$
$x_{4,9} + x_{5,7} \le 1$
$x_{4,9} + x_{5,8} \le 1$

1.4. Type 2 Constraints for the Part ii

$x_{1,0} + x_{2,0} + x_{3,0} + x_{4,0} + x_{5,0}$	≤ 1
$x_{1,0} + x_{1,1} + x_{2,0} + x_{2,1} + x_{3,1} + x_{4,0} + x_{4,1} + x_{5,0} + x_{5,1}$	≤ 1
$x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2} + x_{3,2} + x_{4,0} + x_{4,1} + x_{4,2} + x_{5,0} + x_{5,1} + x_{5,2}$	≤ 1
$x_{1,2} + x_{1,3} + x_{2,2} + x_{2,3} + x_{3,3} + x_{4,1} + x_{4,2} + x_{4,3} + x_{5,0} + x_{5,1} + x_{5,2} + x_{5,3}$	≤ 1
$x_{1,3} + x_{1,4} + x_{2,3} + x_{2,4} + x_{3,4} + x_{4,2} + x_{4,3} + x_{4,4} + x_{5,1} + x_{5,2} + x_{5,3} + x_{5,4}$	≤ 1
$x_{1,4} + x_{1,5} + x_{2,4} + x_{2,5} + x_{3,5} + x_{4,3} + x_{4,4} + x_{4,5} + x_{5,2} + x_{5,3} + x_{5,4} + x_{5,5}$	≤ 1
$x_{1,5} + x_{1,6} + x_{2,5} + x_{2,6} + x_{3,6} + x_{4,4} + x_{4,5} + x_{4,6} + x_{5,3} + x_{5,4} + x_{5,5} + x_{5,6}$	≤ 1
$x_{1,6} + x_{1,7} + x_{2,6} + x_{2,7} + x_{3,7} + x_{4,5} + x_{4,6} + x_{4,7} + x_{5,4} + x_{5,5} + x_{5,6} + x_{5,7}$	≤ 1
$x_{1,7} + x_{1,8} + x_{2,7} + x_{2,8} + x_{3,8} + x_{4,6} + x_{4,7} + x_{4,8} + x_{5,5} + x_{5,6} + x_{5,7} + x_{5,8}$	≤ 1
$x_{1,8} + x_{1,9} + x_{2,8} + x_{2,9} + x_{3,9} + x_{4,7} + x_{4,8} + x_{4,9} + x_{5,6} + x_{5,7} + x_{5,8} + x_{5,9}$	≤ 1
$x_{1,9} + x_{1,10} + x_{2,9} + x_{2,10} + x_{3,10} + x_{4,8} + x_{4,9} + x_{4,10} + x_{5,7} + x_{5,8} + x_{5,9} + x_{5,10}$	≤ 1
$x_{1,10} + x_{1,11} + x_{2,10} + x_{2,11} + x_{3,11} + x_{4,9} + x_{4,10} + x_{4,11} + x_{5,8} + x_{5,9} + x_{5,10} + x_{5,11}$	≤ 1

1.5. Decision Variables

$0 \leq x_{1,0} \leq 1$
$0 \leq x_{1,1} \leq 1$
$0 \leq x_{1,2} \leq 1$
$0 \leq x_{1,3} \leq 1$
$0 \leq x_{1,4} \leq 1$
$0 \leq x_{1,5} \leq 1$
$0 \leq x_{1,6} \leq 1$
$0 \leq x_{1,7} \leq 1$
$0 \leq x_{1,8} \leq 1$
$0 \leq x_{1,9} \leq 1$
$0 \leq x_{1,10} \leq 1$
$0 \leq x_{2,0} \leq 1$
$0 \leq x_{2,1} \leq 1$
$0 \leq x_{2,2} \leq 1$
$0 \leq x_{2,3} \leq 1$
$0 \leq x_{2,4} \leq 1$
$0 \leq x_{2,5} \leq 1$
$0 \leq x_{2,6} \leq 1$
$0 \leq x_{2,7} \leq 1$
$0 \leq x_{2,8} \leq 1$
$0 \leq x_{2,9} \leq 1$
$0 \leq x_{2,10} \leq 1$
$0 \leq x_{3,0} \leq 1$
$0 \leq x_{3,1} \leq 1$
$0 \leq x_{3,2} \leq 1$
$0 \leq x_{3,3} \leq 1$
$0 \leq x_{3,4} \leq 1$
$0 \leq x_{3,5} \leq 1$
$0 \leq x_{3,6} \leq 1$
$0 \leq x_{3,7} \leq 1$
$0 \leq x_{3,8} \leq 1$
$0 \leq x_{3,9} \leq 1$
$0 \leq x_{3,10} \leq 1$
$0 \leq x_{3,11} \leq 1$
$0 \leq x_{4,0} \leq 1$
$0 \leq x_{4,1} \leq 1$
$0 \leq x_{4,2} \leq 1$
$0 \leq x_{4,3} \leq 1$
$0 \leq x_{4,4} \leq 1$
$0 \leq x_{4,5} \leq 1$
$0 \leq x_{4,6} \leq 1$
$0 \leq x_{4,7} \leq 1$
$0 \leq x_{4,8} \leq 1$
$0 \leq x_{4,9} \leq 1$
$0 \leq x_{5,0} \leq 1$
$0 \leq x_{5,1} \leq 1$
$0 \leq x_{5,2} \leq 1$
$0 \leq x_{5,3} \leq 1$
$0 \leq x_{5,4} \leq 1$
$0 \leq x_{5,5} \leq 1$
$0 \leq x_{5,6} \leq 1$
$0 \leq x_{5,7} \leq 1$
$0 \leq x_{5,8} \leq 1$